On the semiclassical theory for universal transmission fluctuations in chaotic systems: the importance of unitarity

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Abstract

The standard semiclassical calculation of transmission correlation functions for chaotic systems is severely influenced by unitarity problems. We show that unitarity alone imposes a set of relationships between cross sections correlation functions which go beyond the diagonal approximation. When these relationships are properly used to supplement the semiclassical scheme we obtain transmission correlation functions in full agreement with the exact statistical theory and the experiment. Our approach also provides a novel prediction for the transmission correlations in the case where time reversal symmetry is present.

I. INTRODUCTION

Since the pioneering work of Blümel and Smilansky [1] the semiclassical S-matrix [2] has been used by many authors to study fundamental questions related to quantum chaotic scattering. In recent years the interest in this subject has grown due to the experimental investigation of electronic transport through small devices, such as open quantum dots [3]. At sufficiently low temperatures, these devices preserve quantum coherence and are called mesoscopic. The conductance in transport processes that preserve quantum coherence is directly related to the S-matrix by the Landauer-Büttiker formula [4]. Since the classical underlying electronic dynamics in quantum dots is believed to be chaotic, these are excellent systems to observe the quantum manifestations of classical chaotic scattering [5].

One of the central issues in mesoscopic physics is to single out statistical universal properties of quantum systems. In this paper we show that this goal cannot be theoretically achieved using the standard semiclassical S-matrix approach, since the latter is not able to provide trustworthy results for universal cross-section or conductance correlation functions. We also show that this situation can be fixed by imposing a set of semiclassical sum rules that guarantee unitarity.

For a device connected to reservoirs by two leads, the Landauer-Büttiker formula relates its conductance G to the transmission coefficient T by the expression $G = (2e^2/\hbar)T$. In electronic transport, T is usually called the dimensionless conductance. When the entrance and exit leads support N_1 and N_2 propagating modes or channels, respectively, the transmission T reads

$$T(x) = \sum_{a=1}^{N_1} \sum_{b=N_1+1}^{N} |S_{ab}(x)|^2 = \sum_{a,b} \sigma_{ab}(x) \ , \tag{1}$$

where we have introduced σ_{ab}, the transition probability (the cross-section, apart from a kinematical factor) from channel b in one lead to channel a in the other lead. The parameter x represents either the energy E, or an external parameter X such as a magnetic field, or both. Without loss of generality we restrict our discussion to the case where $N_1 = N_2 = N/2 \equiv M$.

The original problem of conductance fluctuations in a mesoscopic device is now cast into a more generic one of transmission fluctuations of a quantum process where a particle is chaotically scattered. Thus, this discussion is also of interest for transmission experiments with irregular microwave cavities, "chaotic" atoms and nuclei. Our goal is to use the semiclassical theory to describe the statistical properties of the transmission as the parameter x is varied. This information is contained in the average value of T (a two-point statistical measure of the S-matrix elements) and in its autocorrelation function (a four-point function), defined as

$$(T, T') \equiv \langle T(x) T(x') \rangle - \langle T(x) \rangle \langle T(x') \rangle \ . \tag{2}$$

The average is taken over x and x' keeping the difference $|x - x'|$ fixed. The transmission autocorrelation function is directly related to the covariances of the transition probabilities

$$(T, T') = \sum_{a,c=1}^{M} \sum_{b,d=M+1}^{N} (\sigma_{ab}, \sigma'_{cd}) \ . \tag{3}$$

The variance of T, var(T), is the statistical measure of a fundamental phenomenon in mesoscopic physics: For systems where quantum coherence is preserved the dimensionless conductance displays fluctuations of order unity irrespective of sample size, provided the dynamics is chaotic (or diffusive) and there are no tunneling barriers hindering the transmission. This phenomenon is known as "universal conductance fluctuations" (UCF) [6]. Thus, a successful approximation scheme to explain UCF has to be accurate to the level of unity for the variance of T. In the specific case of quantum dots, *i.e.*, ballistic electronic cavities, the random matrix theory [7,8] and the supersymmetric method [9] are, so far, the successful approaches to calculate var(T).

The purpose of this work is twofold. First we analyze a very simple statistical measure – the average cross section – to show that the standard semiclassical S-matrix theory does not achieve the required precision to be useful as a theory for UCF. In doing this, we indicate its main sources of inaccuracy and discuss the main problems involved in improving the theory. We then show how to fix the inaccuracies by making explicit use of the unitarity of the S

matrix. This procedure is similar in spirit to those used in semiclassical studies of spectra of closed system [10]. It can be viewed as a proposal for a set of semiclassical scattering sum rules to enforce well known exact symmetries of the S-matrix.

II. THE SEMICLASSICAL APPROACH

We start with Miller's semiclassical S-matrix formula [2] (including now transmission and reflections)

$$\widetilde{S}_{ab}(E,X) = \sum_{\mu(a,b)} \sqrt{p_\mu(E,X)}\, e^{i\phi_\mu(E,X)/\hbar} \ , \tag{4}$$

where $\mu(a,b)$ labels the classical trajectories that start at channel b and end at channel a, ϕ_μ are their reduced actions (with a Maslov phase included), and p_μ stands for the classical transition probability of going from a to b through the orbit μ [11] (here and throughout the paper the tilde indicates that the semiclassical approximation is employed). It is implicit in the derivation of Eq. (4) that the number of open channels must be much larger than one and that there are no tunneling barriers between the leads and the cavity. When the scattering is chaotic (and the short time dynamics does not significantly contribute to S) the domains of applicability of the semiclassical theory and random matrix theory coincide and both approaches should be comparable. In this regime the semiclassical approach provides the dynamical explanation for the universality of the scattering fluctuations.

In general the semiclassical S-matrix is not exactly unitary at any given energy E. Indeed, it is only upon energy averaging and for the case of broken time-reversal symmetry (BTRS), that unitarity is automatically fulfilled. We now show this known result so as to present the basic elements and approximations employed in this paper. The energy averaged semiclassical cross section reads

$$\langle \widetilde{\sigma}_{ab} \rangle = \langle |\widetilde{S}_{ab}|^2 \rangle = \sum_{\mu,\nu} \left\langle \sqrt{p_\mu p_\nu}\, e^{i(\phi_\mu - \phi_\nu)/\hbar} \right\rangle \ . \tag{5}$$

Here $\langle \cdots \rangle$ indicates an energy average within an energy window where the classical dynamics presents little changes, nonetheless comprising many resonances. To compute the energy

average one neglects the energy dependence of the probabilities p_μ and uses the diagonal approximation. The latter says that, on average, only orbits having the same action are correlated. If there are no symmetries present, this means that $\langle \exp[i(\phi_\mu - \phi_\nu)/\hbar] \rangle = \delta_{\mu\nu}$. Then

$$\langle \widetilde{\sigma}_{ab} \rangle = \sum_{\mu,\nu} \sqrt{p_\mu p_\nu}\, \delta_{\mu\nu} = \sum_{\mu} p_\mu \ . \tag{6}$$

The proof is completed by using the classical normalization condition [11]

$$\sum_{a=1}^{N} \sum_{\mu(a,b)} p_\mu = 1 \ , \tag{7}$$

which insures that $\sum_a \langle |\widetilde{S}_{ab}|^2 \rangle = 1$.

We shall assume that for any given entrance channel b, all exit channels a are equivalent, *i.e.*,

$$\sum_{\mu(a,b)} p_\mu = \frac{1}{N} \ , \tag{8}$$

which yields

$$\langle \widetilde{\sigma}_{ab} \rangle = \frac{1}{N} \ . \tag{9}$$

The assumption of equivalent channels is justified (in the BTRS case) if the particle typically stays inside the interaction region time enough to be randomized, meaning that it becomes equiprobable to be ejected through any outgoing channel. The analysis of the other limiting case where time reversal symmetry is preserved will be postponed to Section III.

In order to clearly explain why unitarity problems affect the semiclassical theory of transmission fluctuations we introduce the object

$$\widetilde{1}_a \equiv \sum_{b=1}^{N} \widetilde{\sigma}_{ab} = \sum_{b=1}^{N} \sum_{\substack{\mu(a,b) \\ \nu(a,b)}} \sqrt{p_\mu p_\nu}\, e^{i(\phi_\mu - \phi_\nu)/\hbar} \ . \tag{10}$$

If the semiclassical S-matrix had been exact $\widetilde{1}_a = 1$; instead one has $\langle \widetilde{1}_a \rangle = 1$. The lack of precision of the standard semiclassical scattering theory at the four-point level becomes

evident by analyzing the variance $(\widetilde{1}_a, \widetilde{1}_b)$. The semiclassical approximation gives $(\widetilde{1}_a, \widetilde{1}_b) \neq 0$, leading to a "unit fluctuation" problem. To see this, using Eq. (4), we write

$$\left(\widetilde{1}_a, \widetilde{1}_b\right) = \left\langle \sum_{c,d=1}^{N} \sum_{\substack{\mu(a,c) \\ \nu(a,c)}} \sum_{\substack{\mu'(b,d) \\ \nu'(b,d)}} \sqrt{p_\mu p_\nu p_{\mu'} p_{\nu'}} \exp\left[\frac{i}{\hbar}(\phi_\mu - \phi_\nu + \phi_{\mu'} - \phi_{\nu'})\right] \right\rangle - 1 \tag{11}$$

so that the diagonal approximation yields

$$\left(\widetilde{1}_a, \widetilde{1}_b\right) = \sum_{c,d=1}^{N} \sum_{\substack{\mu(a,c) \\ \nu(a,c)}} \sum_{\substack{\mu'(b,d) \\ \nu'(b,d)}} \sqrt{p_\mu p_\nu p_{\mu'} p_{\nu'}} (\delta_{\mu\nu}\delta_{\nu'\mu'} + \delta_{\mu\nu'}\delta_{\nu\mu'}) - 1 \ . \tag{12}$$

The first Kronecker δ product decouples the sums over orbits starting at channel c and ending at a from those entering the scattering region through channel d and exiting through b. The resulting double sum adds up to unity. The second product contains crossed terms which vanish unless $a = b$ and $c = d$. Equation (12) becomes

$$\left(\widetilde{1}_a, \widetilde{1}_b\right) = \delta_{ab} \sum_{c=1}^{N} \sum_{\substack{\mu(a,c) \\ \nu(a,c)}} p_\mu p_\nu = \frac{\delta_{ab}}{N} \ . \tag{13}$$

This inaccuracy is neither unexpected, nor large. However, it has important consequences for the calculation of transmission fluctuations. This becomes evident by inspecting

$$\left(\sum_{a=1}^{N} \widetilde{1}_a, \sum_{b=1}^{N} \widetilde{1}_b\right) = \left(\sum_{a,c=1}^{N} \widetilde{\sigma}_{ac}, \sum_{b,d=1}^{N} \widetilde{\sigma}_{bd}\right) = 1 \neq 0 \tag{14}$$

which has the same double sum structure of the transmission variance and shows an inaccuracy exactly of the order of the effect that we aim to describe.

Let us be more explicit and go back to the analysis of the transmission autocorrelation function. Recalling Eq. (3) and assuming channels to be statistically equivalent we write

$$(T, T') = M^2(\sigma_{ab}, \sigma'_{ab}) + 2M^2(M-1)(\sigma_{ab}, \sigma'_{ac}) + M^2(M-1)^2(\sigma_{ab}, \sigma'_{cd}) \ . \tag{15}$$

Here we use the convention that different indices a and b in the covariances imply that $a \neq b$. This means that for the above equation $b \neq c$ in the second term of its RHS, and $a \neq c$ and $b \neq d$ for the third one. We shall demonstrate below that, owing to unitarity, all three terms in the RHS of Eq. (15) are of the same order of magnitude. However, within the

diagonal approximation, both the semiclassical covariances $(\tilde{\sigma}_{ab}, \tilde{\sigma}'_{ac})$ and $(\tilde{\sigma}_{ab}, \tilde{\sigma}'_{cd})$ are zero. Let us admit that the semiclassical approach gives the correct result for the transmission probability covariances to order $1/N^2$. Then, for a successful description of the transmission fluctuations, the theory has to be improved to access the first non vanishing order in the non diagonal terms $(\sigma_{ab}, \sigma'_{ac})$ and $(\sigma_{ab}, \sigma'_{cd})$, which are $\mathcal{O}(N^{-3})$ and $\mathcal{O}(N^{-4})$, respectively. Though desirable this is not really necessary. The alternative scheme we propose is to bypass the explicit semiclassical calculation of the nondiagonal covariances and use the unitarity of the S matrix to relate the latter covariances to the diagonal one. Having expressed (T, T') in terms of $(\sigma_{ab}, \sigma'_{ab})$ alone, we use the semiclassical approximation only at the very end.

III. ENFORCING UNITARITY

The relations among diagonal and nondiagonal covariances can be easily obtained from

$$(\sum_{b=1}^{N} \sigma_{ab}, \sigma'_{cd}) = (1, \sigma'_{cd}) = 0 \; , \tag{16}$$

which follows from unitarity. To proceed further we have to separately analyze the cases where either time reversal symmetry is absent (BTRS) or present (TRS). This distinction is necessary because the "elastic" processes ($a = b$) and the "inelastic" ones ($a \neq b$) display different statistical properties when time reversal symmetry is preserved. Indeed it is well known that due to quantum interference, time reversal symmetry enhances the average reflection probability $\langle \sigma_{aa} \rangle$ by a factor of two [12]. This can be understood semiclassically by noting that in the TRS case there are pairs of orbits having the same action (time reversal partners) which contribute to σ_{aa}, thus interfering constructively to produce the factor two. Due to this effect the classical equivalence of channels breaks down at the quantum level. In this case the equivalence is only restored when time reversal symmetry is broken, leading to the considerations presented in Section II.

To illustrate the consequences of this phenomenon and the spirit of our scheme, let us analyze $\langle T(x) \rangle$ in the crossover regime from TRS to BTRS. Recall that

$$\langle T(x)\rangle = \sum_{a=1}^{M}\sum_{b=M+1}^{N}\langle\sigma_{ab}(x)\rangle = M^2\langle\sigma_{ab}(x)\rangle. \tag{17}$$

Here x parameterizes a Hamiltonian change breaking time reversal symmetry as x grows from zero (TRS) to some critical value x^* (BTRS). Unitarity relates diagonal and off-diagonal averages:

$$1 = \langle\sigma_{aa}(x)\rangle + (N-1)\langle\sigma_{ab}(x)\rangle\ . \tag{18}$$

This equation allows us to write $\langle T\rangle$ in terms of $\langle\sigma_{aa}\rangle$, the average that is semiclassically sensitive to time reversal effects, to obtain

$$\langle T(x)\rangle = \frac{N^2}{4(N-1)}(1-\langle\sigma_{aa}(x)\rangle)\ . \tag{19}$$

For $x=0$ the elastic enhancement is maximal and hence $\langle T(x)\rangle$ takes its smallest value. In mesoscopic physics, to distinguish from strong localization which is a phenomenon very different in origin, the reduction of transmission due to TRS is called the weak localization peak. Up to this point Eq. (19) is an exact expression. The semiclassical result is obtained by calculating $\langle\sigma_{aa}(x)\rangle$ from Miller's formula. For x representing a magnetic field, the semiclassical approach gives a Lorentzian shape for the weak localization peak [13] in agreement with RMT [14]. The amplitude of the weak localization correction can be readily obtained recalling that $\langle\widetilde{\sigma}_{aa}(0)\rangle \approx 2/N$ and $\langle\widetilde{\sigma}_{aa}(x\geq x^*)\rangle \approx 1/N$, so that

$$\langle\widetilde{T}(0)\rangle - \langle\widetilde{T}(x^*)\rangle = -\frac{1}{4}\ , \tag{20}$$

again in agreement with random matrix theory [7]. The discussion above is not entirely original and was inspired by the pioneer semiclassical study of the weak localization peak in ballistic cavities developed by Baranger and collaborators [13]. Based on the same strategy presented above, we are now ready to understand the UCF problem.

A. Transmission fluctuations in systems with broken time reversal symmetry

In this case all S matrix elements are statistically equivalent. In order to express (T,T') in terms of $(\sigma_{ab},\sigma'_{ab})$ it suffices to consider the following two independent unitarity equations

$$\sum_{b=1}^{N}(\sigma_{ab},\sigma'_{ac}) = 0\;, \quad \sum_{b=1}^{N}(\sigma_{ab},\sigma'_{cd}) = 0 \tag{21}$$

($c \neq a$ in the second equation). The above relations can be reduced to:

$$(\sigma_{ab},\sigma'_{ab}) + (N-1)(\sigma_{ab},\sigma'_{ac}) = 0\;, \quad (\sigma_{ab},\sigma'_{ac}) + (N-1)(\sigma_{ab},\sigma'_{cd}) = 0 \tag{22}$$

(the convention about indices being the same as in Eq. (15)). Notice that these relations are not satisfied in the diagonal approximation. At the semiclassical level Eqs. (21) and (22) can be regarded as sum rules that go beyond the diagonal approximation. Insertion of Eqs. (22) into Eq. (15) readily renders

$$(T,T') = \frac{M^4}{(2M-1)^2}(\sigma_{ab},\sigma'_{ab})\;. \tag{23}$$

As in the two-point analysis this equation is exact.

Now we are ready to employ the semiclassical approximation to compute $(\sigma_{ab},\sigma'_{ab})$. Let us first consider the case where x stands for the energy, *i.e.*,

$$(\sigma_{ab},\sigma'_{ab}) = C_{ab}(\varepsilon) \equiv \left\langle \sigma_{ab}(E+\frac{\varepsilon}{2})\; \sigma_{ab}(E-\frac{\varepsilon}{2})\right\rangle_E - \langle\sigma_{ab}\rangle_E^2\;. \tag{24}$$

The semiclassical autocorrelation function $\widetilde{C}_{ab}(\varepsilon)$ can be calculated for classically small values of ε, *i.e.*, for energy differences such that the classical perturbation theory holds. In this case one keeps the stability coefficients constant and expands the actions to first order in ε, *i.e.*, $\phi_\mu(E\pm\varepsilon/2) \approx \phi_\mu(E) \pm \tau_\mu\varepsilon/2$; here τ_μ is the time the particle takes to travel from channel b to channel a along the orbit μ. After the diagonal approximation we obtain

$$\left\langle \widetilde{\sigma}_{ab}(E+\frac{\varepsilon}{2})\; \widetilde{\sigma}_{ab}(E-\frac{\varepsilon}{2})\right\rangle = \sum_{\substack{\mu(a,b)\\ \nu(a,b)}}\;\sum_{\substack{\mu'(a,b)\\ \nu'(a,b)}} \sqrt{p_\mu p_\nu p_{\mu'} p_{\nu'}}\,(\delta_{\mu\nu}\delta_{\nu'\mu'} + \delta_{\mu\nu'}\delta_{\nu\mu'}) \\ \times \exp\left[\frac{i\varepsilon}{2\hbar}\left(\tau_\mu - \tau_\nu + \tau_{\mu'} - \tau_{\nu'}\right)\right]\;. \tag{25}$$

Using the same arguments employed after Eq. (12) we arrive at

$$\widetilde{C}_{ab}(\varepsilon) = \sum_{\substack{\mu(a,b)\\ \nu(a,b)}} p_\mu p_\nu \exp\left[i\frac{\varepsilon}{\hbar}(\tau_\mu - \tau_\nu)\right]\;. \tag{26}$$

According to the analogue of the Hannay-Ozorio de Almeida sum rule for open systems [15],

$$\sum_{t \le \tau_\mu \le t+\delta t} p_\mu = \frac{\gamma}{N} e^{-\gamma t} \delta t \ , \tag{27}$$

where $\sum_{t \le \tau_\mu \le t+\delta t} p_\mu$ is the sum of all classical transition probabilities from channel a to b through trajectories within a small time interval $[t, t+\delta t]$, where δt is classically small. The exponential is determined by the inverse escape time γ, later to be associated to an energy width $\Gamma = \hbar\gamma$. Replacing the sum over orbits by an integral over the time, we finally obtain

$$\widetilde{C}_{ab}(\varepsilon) = \frac{1}{N^2} \frac{1}{1+(\varepsilon/\Gamma)^2} \ . \tag{28}$$

Before commenting on this result, we shall generalize it by accounting for an external parametric change X in the Hamiltonian. For instance, in (mesoscopic physics) experiments X is frequently an external magnetic field. The strategy to compute the generalized correlation function is, as above, based on classical perturbation theory. By expanding the reduced action to first order in X, we have to deal with $Q_\mu \equiv \partial\phi_\mu/\partial X$. A full account of the technical details involved in calculating the parametric correlation can be found in Ref. [16]. The basic step though is to compute the time average

$$\left\langle e^{iQ_\mu \delta X/\hbar} \right\rangle_{\delta t} = \exp\left[-\frac{\delta X^2}{2\hbar^2} \langle Q^2(t) \rangle_{\delta t} \right] \ . \tag{29}$$

Since $\langle Q^2(t) \rangle_{\delta t}$ grows diffusively with time, *i.e.*, $\langle Q^2(t) \rangle_{\delta t} = \alpha t$, the autocorrelation function becomes

$$\widetilde{C}_{ab}(\varepsilon, \delta X) = \frac{1}{N^2} \frac{1}{[1+(\delta X/X_c)^2]^2 + (\varepsilon/\Gamma)^2} \ , \tag{30}$$

with

$$X_c^2 \equiv 2\hbar\Gamma/\alpha \ . \tag{31}$$

Then the semiclassical result that includes unitarity restrictions is

$$(\widetilde{T}, \widetilde{T}') = \frac{1}{16} \frac{1}{[1+(\delta X/X_c)^2]^2 + (\varepsilon/\Gamma)^2} + \mathcal{O}(1/N) \ . \tag{32}$$

Remarkably, this result agrees exactly with the dimensionless conductance autocorrelation function for open ballistic dots in the limit $N \gg 1$ obtained by Efetov [9] using the supersymmetric technique. The agreement extends also to the structure of the parameter X_c (31) if one relates α to the level velocity of closed systems as defined in Ref. [16]. In this respect the semiclassical approach is complementary to random matrix theories, since it provides a dynamical interpretation for the nonuniversal quantities X_c and Γ.

B. Systems with time reversal symmetry

Now the S matrix is symmetric and the statistical properties of diagonal and off-diagonal elements are different. Hence we need to write down two additional unitarity relations, as compared with the BTRS case, in order to single out the elastic case separately:

$$\sum_{b=1}^{N}(\sigma_{ab}, \sigma'_{aa}) = 0 \ , \quad \sum_{b=1}^{N}(\sigma_{cb}, \sigma'_{aa}) = 0 \ , \quad \sum_{b=1}^{N}(\sigma_{ab}, \sigma'_{ac}) = 0 \ , \quad \sum_{b=1}^{N}(\sigma_{ab}, \sigma'_{cd}) = 0 \tag{33}$$

$(c, d \neq a)$. In terms of the basic covariances, the above system of equations is rewritten as

$$\begin{aligned}
(\sigma_{aa}, \sigma'_{aa}) + (N-1)(\sigma_{aa}, \sigma'_{ab}) &= 0 \\
(\sigma_{aa}, \sigma'_{ab}) + (\sigma_{aa}, \sigma'_{bb}) + (N-2)(\sigma_{aa}, \sigma'_{bc}) &= 0 \\
(\sigma_{ab}, \sigma'_{ab}) + (\sigma_{aa}, \sigma'_{ab}) + (N-2)(\sigma_{ab}, \sigma'_{ac}) &= 0 \\
(\sigma_{aa}, \sigma'_{bc}) + 2(\sigma_{ab}, \sigma'_{ac}) + (N-3)(\sigma_{ab}, \sigma'_{cd}) &= 0 \ ,
\end{aligned} \tag{34}$$

where the channel index convention is that following Eq. (15). The only difference with the BTRS case is the factor 2 in the last equation, which is a consequence of the symmetry of S. As in the preceding subsection, we would like to express all covariances in terms of $(\sigma_{aa}, \sigma'_{aa})$ and $(\sigma_{ab}, \sigma'_{ab})$ which are the only nonzero ones in the diagonal approximation. Regrettably, we have more unknowns than equations, and it is not possible to obtain an exact equation like Eq. (23). However, as we are only interested in a relation which is correct to leading order in $1/N$, it suffices to consider the simplified system

$$(\sigma_{ab}, \sigma'_{ab}) + N(\sigma_{ab}, \sigma'_{ac}) = \mathcal{O}(N^{-3}) \ , \quad 2(\sigma_{ab}, \sigma'_{ac}) + N(\sigma_{ab}, \sigma'_{cd}) = \mathcal{O}(N^{-4}) \ , \tag{35}$$

which is obtained from the last two equations in (34) by keeping only the leading terms. These relations are the TRS analogues of (22) and lead to

$$(T, T') \approx \frac{M^2}{2}(\sigma_{ab}, \sigma'_{ab}) \; . \tag{36}$$

Semiclassically, time reversal effects only manifest themselves in the diagonal covariances $(\sigma_{aa}, \sigma'_{aa})$. The correlator $(\widetilde{\sigma}_{ab}, \widetilde{\sigma}'_{ab})$ is the same as that for the BTRS case. Thus, at the semiclassical level, the effect of time reversal symmetry is to enhance the transmission fluctuations by a factor of two (cf. Eq. (23)), without changing the shape of the correlation function, that is

$$(\widetilde{T}, \widetilde{T}') = \frac{1}{8}\frac{1}{[1 + (\delta X/X_c)^2]^2 + (\varepsilon/\Gamma)^2} + \mathcal{O}(1/N) \; . \tag{37}$$

We are not aware of any study of the transmission autocorrelation function for the TRS case using the supersymmetric method. For technical reasons it is much simpler to obtain the variance $\mathrm{var}(T)$ using random matrix theory. This result is known [7] and, for $N \gg 1$, agrees with our semiclassical calculation

$$\mathrm{var}(\widetilde{T}) = \frac{1}{8} \; . \tag{38}$$

IV. CONCLUDING REMARKS

Spectral studies in closed system have shown that the diagonal approximation should start to fail when the orbits involved have periods of the order of the Heisenberg time τ_H. In scattering systems, the contribution of orbits with periods larger than the mean escape time τ_e is negligible. Given that in the semiclassical regime $\tau_e \ll \tau_H$ it is generally accepted that the diagonal approximation should be unproblematic for scattering systems.

However, we have shown that the standard semiclassical approach fails to describe the transmission fluctuations because the diagonal approximation does not preserve the unitarity of the S matrix to the required precision. One way to circumvent this problem, perhaps the most satisfactory from a theoretical point of view, is to improve the semiclassical theory

to include correlations between different orbits. Alternatively, we have shown that the unitarity of the S matrix can be used to express the transmission autocorrelation function in terms of transmission probabilities. Such expression contains some information about unitarity allowing the standard semiclassical approximation to be invoked resulting in a theory consistent with UCF.

Other difficulties are also encountered when calculating the transmission correlations: The semiclassical transmission correlator is not translationally invariant. For instance, this is manifest in the fact that

$$\langle \widetilde{T}(E)\widetilde{T}(E+\varepsilon)\rangle \neq \langle \widetilde{T}(E+\varepsilon/2)\widetilde{T}(E-\varepsilon/2)\rangle \ , \tag{39}$$

which can easily be checked by inspection. In our calculations we preferred to use $\langle \widetilde{T}(E+\varepsilon/2)\widetilde{T}(E-\varepsilon/2)\rangle$ because it is explicitly real. This choice is consistent with the spirit of this work, *i.e.*, all information about exact quantum symmetries must be used in trying to compensate for the shortcomings of the semiclassical S matrix.

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